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Topics In String Phenomenology[★]

MICHAEL DINE

*Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064*

Abstract

We consider two questions in string “phenomenology.” First, are there any generic string predictions? Second, are there any general lessons which string theory suggests for thinking about low energy models, particularly in the framework of supersymmetry? Among the topics we consider are the squark and slepton spectrum, flavor symmetries, discrete symmetries including CP , and Peccei-Quinn symmetries. We also note that in some cases, discrete symmetries can be used to constrain the form of supersymmetry breaking.

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1. Introduction

The title of this talk is rather presumptuous. In my view, we are far from possessing any string phenomenology; we still have very little idea how the observable world might emerge from string theory. This is not to minimize the fact that many compactifications of string theory have been discovered with desirable features: standard model gauge groups, three generations of quarks and leptons, low energy supersymmetry, intricate patterns of discrete symmetry and more. Yet in detail, it is probably safe to say that all of these models have serious flaws. Moreover, we don't presently have a clue as to how or why one of these models might be picked out over another or why the cosmological constant should vanish after supersymmetry breaking.

As a result, my goals today will be far more modest. I won't review the many interesting efforts to develop a detailed string phenomenology. Rather I will

1. Look for features of string compactifications which might be generic, leading to qualitative or quantitative predictions
2. Seek insights from string theory into more conventional model building.

Virtually all of what I have to say will be in the framework of models with low energy supersymmetry. In the first category, I will consider the minimal supersymmetric standard model. This model has become a paradigm for a low energy supersymmetric world. Yet it rests on a set of strong assumptions about the underlying microscopic theory. Generically, these assumptions are not true in string theory. Recently, however, Kaplunovsky and Louis have noted that there is one scenario for string dynamics in which they *are* true.^[1] In this scenario, one obtains a two (or three) parameter description of the low energy world, and predicts a significant degree of degeneracy among squarks and sleptons (necessary to suppress rare processes). We will also describe alternative, string-inspired scenarios for obtaining squark degeneracy based on flavor symmetries.^[2,3,4]

Next I will turn to a variety of questions in the second category, mostly involving symmetries. Recently there has been renewed discussion of the plausibility of global symmetries, both continuous and discrete. In string theory, it has been known for some time that there are no (unbroken) continuous global symmetries.^[5] String theory exhibits intricate patterns of discrete symmetries, and it is often conjectured that these symmetries are also gauge symmetries. We will see, however, that this is not always the case. String perturbation theory often exhibits anomalous, global discrete symmetries. Such symmetries could be of great phenomenological importance; they could, for example, lead to $m_u = 0$. (Other scenarios, with stringy features, which lead naturally to $m_u = 0$ have been considered in ref. 6.) These symmetries are closely related to certain gauged,

non-anomalous discrete symmetries. These gauge symmetries are spontaneously broken; the model-independent axion transforms under them non-linearly. As a result, they would seem to be rather uninteresting. In fact, however, we will see that using these symmetries, one can hope to restrict the form of any non-perturbative superpotential which might be generated in string theory. Indeed, we will see that under these circumstances, one can argue (subject to some plausible assumptions) that “gluino condensation” is the largest supersymmetry-violating effect at weak coupling. These considerations may bear on the possibility of a “duality symmetry” involving the dilaton, discussed by Joanne Cohn and John Schwarz at this meeting,^[7] as well as on the general question of “stringy non-perturbative effects.”^[8]

Finally, we will comment on CP and the strong CP problem. We will see, first, that CP is a discrete gauge symmetry in string theory (it can be thought of as a combination of a general coordinate transformation and a non-Abelian gauge transformation in the higher dimensional space). As a result, there can be no non-perturbative CP -violating parameters. CP -violation is necessarily spontaneous, and any CP -violation is in principle, calculable. It is probably premature to develop a detailed theory of the origin of the KM angle in string theory. But it is of interest to consider how the strong CP problem might be resolved in string theory. We have already remarked that string theory may be a framework in which to understand $m_u = 0$. We will comment on various aspects of axions in string theory, particularly in light of recent criticisms of the axion idea.^[9] We will also see that, quite generally in the framework of supersymmetric models (not just strings), it is difficult to implement scenarios of the Nelson-Barr type, in which θ is arranged to be zero at tree level.^[10,11] The problem is that generically radiative corrections to θ are unacceptably large, unless the soft breakings satisfy certain striking constraints.^[2]

2. Supersymmetry Breaking and the Problem of the Dilaton

It is appropriate to begin by reviewing the suggestions which have been made for how supersymmetry breaking might arise in string theory. These revolve large around the dynamics of “gaugino condensation” in some hidden sector gauge group.^[12] The basic idea is very simple. Suppose that below the Planck scale the model contains a gauge group, \mathcal{G} , under which none of the matter fields transform. The scale of the group is

$$\Lambda \sim e^{-\frac{8\pi^2}{g^2 b_0}}. \quad (2.1)$$

At this scale one expects that gluinos condense, i.e.

$$\langle \lambda\lambda \rangle \propto \Lambda^3. \quad (2.2)$$

In the low energy theory, the dilaton superfield, S , couples universally to the gauge fields. In particular, the auxiliary component of S , F_S , couples to $\lambda\lambda$, so one finds a potential

$$V \sim |\langle \lambda\lambda \rangle|^2. \quad (2.3)$$

However, in string theory, the coupling, g , is itself determined by the expectation value of the dilaton, $g \sim e^{-D}$. Thus V is a potential for the dilaton. This potential can be understood as arising from a non-perturbatively generated superpotential of the form

$$W \sim e^{-S/b_0} \quad (2.4)$$

where $S = e^{-D} + ia$, a being the axion field.

Unfortunately, however, while the appearance of this superpotential constitutes dynamical supersymmetry breaking, at least at weak coupling, this potential has no minimum except at zero coupling, i.e. infinite value of the dilaton vev.^[13] One can object that perhaps this problem is an artifact of our assumptions. Indeed, except in special cases (to be discussed below) there is no argument that high energy, non-perturbative string effects cannot be *larger* than any effects which can be seen in the low energy theory. For example, integrating out massive string modes might lead to a superpotential with a different dependence on the dilaton than that of eqn. (2.4). A little thought, however, makes clear that the problem of the runaway dilaton is generic; the potential will always go to zero in weak coupling. One can hope that there is a minimum at strong coupling. Such a prospect, however, is disturbing, not merely because we have at present no idea how to treat strongly coupled string theories, but also because it is precisely the features of string theory at weak coupling which make it so attractive.

There has been at least one interesting proposal to remedy this problem.^[14] Suppose one has several hidden sector groups, each producing a gluino condensate. Then, for some choices of phase of the condensate, the non-perturbative superpotential has the form

$$W = \alpha e^{-aS} - \beta e^{-bS}. \quad (2.5)$$

Now, because of the negative sign, one can have a local minimum in the potential. If $a \approx b$, then it is possible that this minimum lies at large S , i.e. weak coupling. Note, however, that there is no parameter at our disposal which can be made arbitrarily small. Instead, one tries to find compactifications which lead to numerically large values of S , values which, from a field-theoretic perspective, one expects to

correspond to weak coupling.* In practice, examples have been constructed with rather small effective coupling. However, even though supersymmetry turns out to be broken in some of these minima, the cosmological constant is typically non-zero even in the leading approximation.

3. The Minimal Supersymmetric Standard Model

The MSSM, as noted in the introduction, has become the paradigm for a supersymmetric model of nature. In this section, I would like to review the features of this model. We will see that the basic assumptions of the model are very strong, but that if supersymmetry breaking in string theory takes one very particular form, string theory can yield precisely such a theory.

The particle content of the MSSM is that of a supersymmetrized ordinary standard model with two Higgs doublets (i.e. one takes the gauge group to be $SU(3) \times SU(2) \times U(1)$, introduces a chiral superfield for every quark and lepton, and introduces two $SU(2)$ doublet fields, H_1 and H_2). Of course, in string models one typically obtains additional fields, such as gauge singlet chiral fields and additional gauge interactions, but such a spectrum might emerge from a string model.

To specify the model, it is also necessary to make some statements about soft breakings. There are a variety of experimental constraints on these parameters. Obviously, the masses of the superpartners must be large enough that these particles not have been seen. Presumably, these masses should not be arbitrarily large, since in that case it will be necessary to fine tune parameters in order to obtain electroweak breaking. Operationally, the most severe constraint of this type arises from corrections to the H_1 mass containing a top squark; these are proportional to $3m_{\tilde{t}}^2 g_t^2 / (16\pi^2)$ times a logarithmic factor. Here g_t is the top quark Yukawa coupling, and $m_{\tilde{t}}^2$ is the top squark mass. One also requires significant degeneracy of squark masses to avoid flavor changing neutral currents (FCNC's). Indeed, from the real part of the $K - \bar{K}$ mass difference, one obtains a constraint (ref. 15)

$$\frac{\delta m^2}{m_{susy}^2} < 10^{-2} - 10^{-3} \quad (3.1)$$

* One can debate whether, in the absence of a small parameter which can formally be taken to zero, there can be a sense in which weak coupling is valid. To fully justify this procedure one must be able to argue that even though the effects of the high energy theory are not under control, one can still integrate them out, obtaining an effective action of a known form, and that the largest supersymmetry breaking effects are those which are visible in the low energy theory. In the case of anomalous discrete symmetries, discussed below, this may not be necessary.

where δm^2 represents a typical splitting and m_{susy} a typical susy-breaking mass.

Without specifying further details of the soft breakings, it should be noted that this model has already scored some striking successes.^[16] Perhaps most dramatic is the successful unification of gauge (and some Yukawa) couplings which results in this picture.^[17] Other good features include the presence of suitable dark matter candidates.

Of course, the model as it stands possesses a large number of free parameters, and a number of theoretical approaches have been adopted to narrowing this parameter space. The most common assumption is that all of the squarks and sleptons are degenerate at the unification scale, and that the soft-breaking cubic couplings are simply proportional to the superpotential. One then evolves to low energies using the renormalization group, trading one parameter for M_Z . This leaves four parameters (excluding m_t). Further constraints which are often imposed include: $m_b = m_\tau$ at the high scale; presence of a suitable dark matter candidate, and absence of fine tuning. Such programs lead to suggestive values of the soft-breaking parameters.^[16]

These are strong assumptions, and they are not easy to justify within the most popular framework for supersymmetry model building: supergravity theories with supersymmetry broken in a hidden sector. In such models one has a set of “hidden sector fields,” Φ and “visible sector fields,” Q^I (quarks, leptons, etc.; here we are adopting the notation of ref. 1). The general theory of this type is specified by three functions: the Kahler potential, $K(\phi, \phi^*)$, the superpotential, $W(\phi)$, and a function $f(\phi)$ which describes the coupling of matter to gauge fields. In this approach, one assumes that some of the hidden sector auxiliary fields obtain vacuum expectation values, $F_\Phi \sim m_{3/2} M_p$. Taylor expanding the Kahler potential about the origin of the visible sector fields,

$$K = K(\Phi^*, \Phi) + Z_{\bar{I}J}(\Phi^*, \Phi) Q^{*\bar{I}} Q^J + \left(\frac{1}{2} H_{IJ} Q^I Q^J + cc.\right) + \dots \quad (3.2)$$

The term $Z_{\bar{I}J}$ is the origin of the squark masses. Squark degeneracy means

$$Z_{\bar{I}J} \propto \delta_{\bar{I}J}. \quad (3.3)$$

But there is no reason, in general, for this condition to hold; there is certainly no symmetry of the low energy theory which enforces it. There is no reason one would expect this to hold generically in string theory; indeed, Ibanez and Lust^[18] have shown in particular orbifold examples that such a relation does not hold.

While a number of suggestions have been made over the years to understand degeneracy, Kaplunovsky and Louis have recently pointed out a possible stringy

solution.^[1] These authors tried to study the question of soft breakings in the context of string theory, without making detailed assumptions about the origin of supersymmetry breaking. They assumed only

1. The potential has a stable minimum at reasonably weak coupling.
2. $V \approx 0$ at the minimum
3. SUSY breaking is primarily due to expectation values either for the moduli or the dilaton, $\langle F_M \rangle$ or $\langle F_S \rangle$.

These assumptions are modest in the sense that they are probably the minimal assumptions required for any plausible phenomenology; whether or not these conditions are actually achieved in string theory is another matter. Without further assumptions, one can make only modest statements (see V. Kaplunovsky's talk at this meeting). In particular, there is no explanation for squark degeneracy. However, these authors noted that if one assumes $\langle F_S \rangle \gg \langle F_M \rangle$, then, because of the universality of the dilaton couplings, one *does* obtain a significant degree of squark and slepton degeneracy, and one in fact obtains a highly predictive scenario. Examining the tree level lagrangian, one finds that the gaugino masses, m_g , the squark and slepton masses, m_ϕ^2 , and the A parameter (the coefficient of the cubic soft breaking terms in the potential) are given by

$$m_g = \frac{\sqrt{3}}{2} m_{3/2} \quad m_\phi^2 = m_{3/2}^2 \quad A = -\sqrt{3} m_{3/2}. \quad (3.4)$$

This is precisely the structure of the soft-breaking parameters in the minimal supersymmetric standard model! Moreover, instead of the four parameters listed above, there is now only one (or two, depending on what one assumes about the origin of the so-called μ parameter).^[1] One expects that these relations will be corrected by one loop effects of order α_{GUT}/π . Whether or not this is good enough to explain the suppression of FCNC's will depend on the precise value of the one loop coefficients. Naively, however, one does not expect these corrections to be small enough, since at the large scale α_s/π , for example, is likely to be larger than 10^{-2} , so something more is likely to be required, particularly to understand the smallness of the imaginary part of $K\bar{K}$. Still, this looks tantalizingly close.

We do not know, of course, whether string theory dynamics satisfy this condition. Kaplunovsky and Louis, in fact, argue that they do not for known susy breaking schemes. However, we know very little about susy breaking in string theory; no known mechanism even satisfies the modest set of assumptions listed above. Given that the dilaton breaking scenario looks close to what one wants, and that only one assumption is required, it is interesting to explore its consequences. The required renormalization group analysis has been performed in ref.

19. Assuming MSSM particle content, one finds that it is difficult to implement this scenario given current experimental constraints; significant fine tunings (at the part in 10^{-2} level) are required. Non-minimal models are presumably not so highly constrained (and of course not so predictive); it will be of interest to explore their phenomenology.

4. Flavor Symmetries: An Alternative Solution to the Degeneracy Problem

An alternative approach to the problem of degeneracy, about which string theory also offers some suggestive clues, is to assume that there is some underlying flavor symmetry.^[2,6] One possibility is that there exists a non-Abelian, gauged horizontal symmetry. The obvious problem with this proposal is that whatever flavor symmetry there is must be badly broken in order to explain the ordinary quark mass matrix. The simplest model which one can use to examine this question possesses an extra gauge symmetry $SU(2)_H$. Under this symmetry we assume that the quarks transform as doublets and singlets: $Q^a, \bar{u}^a, \bar{d}^a$ and singlets, Q_s, u_s, \bar{d}_s and similarly for the leptons. The Higgs fields are assumed to be $SU(2)_H$ singlets.

In addition, to break the horizontal symmetry, we assume that we have some $SU(2)_H$ doublets which are standard model singlets, $\Phi_{(i)}^a$. From a stringy perspective, the presence of such particles is quite plausible. We might imagine that the $SU(2)_H$ symmetry corresponds to an enhanced gauge symmetry at some particular point in the moduli space. The fields $\Phi_{(i)}^a$ then represent moduli. Giving them expectation values corresponds to moving away from the special point. If this identification is correct, these fields have no potential, and can easily obtain vev's of order M_p . For what follows, we will assume

$$\frac{\langle \Phi_{(i)}^a \rangle}{M_p} \sim 10^{-1}. \quad (4.1)$$

Such an assumption is not unnatural. Vev's of this order might arise in the presence of Fayet-Iliopoulos terms generated at one loop, for example.

To proceed, we need to adopt a set of rules about the sizes of various couplings. We will enforce 't Hooft's notion of naturalness:^[20] couplings can be small only if the theory becomes more symmetric in that limit. Ultimately, we would like to explain such small parameters through additional symmetries (e.g. discrete symmetries of the type to be discussed shortly).

Let us consider, then, the allowed couplings in the lagrangian. The superpotential just below M_p contains dimension-four terms:

$$W_q = \lambda_1 \epsilon_{ab} Q_a \bar{d}_b H_1 + \lambda_2 \epsilon_{ab} Q_a \bar{u}_b H_2 + \lambda_3 Q_s \bar{d}_s H_1 + \lambda_4 Q_s \bar{u}_s H_2. \quad (4.2)$$

These give rise to $SU(2)_H$ symmetric terms in the mass matrix. Clearly we need to assume that λ_1 and λ_2 are small (this might be arranged by means of a discrete symmetry). $SU(2)_H$ -violating terms arise at the level of dimension five and dimension six operators:

$$\frac{1}{M_p} (\lambda_5^i \epsilon_{ab} \Phi_a^i Q_b \bar{d}_s H_1 + \lambda_6^i \epsilon_{ab} \Phi_a^i Q_s \bar{d}_b H_1) + \frac{1}{M_p^2} (\lambda_7^{ij} \epsilon_{ab} \epsilon_{cd} \Phi_a^i \Phi_c^j Q_b \bar{d}_d H_1 + \dots). \quad (4.3)$$

Note that the charmed-quark mass must arise from these operators, and is thus of order $(\Phi/M_p)^2$, so Φ/M_p can't be much smaller than 0.1.

The breaking of the squark degeneracy can also be understood in terms of the effective action at scales slightly below M_p . This lagrangian contains dimension-four, soft-breaking terms which give $SU(2)_H$ -symmetric contributions to the squark mass matrices:

$$V_{soft} = m_1^2 |Q_a|^2 + m_2^2 |Q_s|^2 + m_3^2 |\bar{u}_a|^2 + m_4^2 |\bar{u}_s|^2 + \dots \\ + A_1 \lambda_1 Q \bar{d} H_1 + A_2 \lambda_2 Q \bar{u} H_1 + \dots + h.c. \quad (4.4)$$

Here, m_i and A_i are of order m_{susy} . Breaking of the symmetry will arise through terms of the type

$$\delta V_{soft}^2 = \frac{m_{susy}^2}{M_p} (\gamma_1 \Phi_1 Q Q_s^* + \dots) + \frac{m_{susy}^2}{M_p^2} (\gamma'_1 \Phi_1 Q \Phi_2 Q^* + \dots) \quad (4.5)$$

and

$$\delta V_{soft}^3 = \frac{m_{susy}}{M_p} \lambda_5^1 Q \bar{d}_s H_1 (\eta_1 \Phi_1 + \eta_2 \Phi_2 + \eta_3 \Phi_2^*) \\ + \frac{m_{susy}}{M_p^2} \lambda_7^{11} Q \bar{d} H_1 (\eta'_1 \Phi_1 \Phi_1 + \eta'_2 \Phi_1 \Phi_2 + \eta'_3 \Phi_1 \Phi_2^*) + \dots \quad (4.6)$$

We have omitted $SU(2)_H$ indices on Q , \bar{u} , \bar{d} but terms with all possible contractions should be understood. Here γ , γ' , η and η' are dimensionless numbers. By 't Hooft's naturalness criterion,^[20] many of these couplings should not be much less than one; the theory does not become any more symmetric if these quantities vanish. As a result, the generic symmetry-violating terms in the first two generations are of order $(\Phi/M_p)^2 \sim 10^{-2}$. Some of these couplings, however, can (and should!) naturally be small. So there is no difficulty with suppressing FCNC's.

This framework is predictive. For example, it suggests that there should be a high degree of degeneracy only in the first two squark and slepton generations. The small value of the neutron electric dipole moment is also readily accommodated here. Recently, Seiberg and Pouliot have considered models in which non-Abelian symmetries play a role both in producing squark degeneracy and in explaining the features of the quark mass matrix. Such models are clearly of great interest.^[4]

An alternative approach to the problem of fnc's in supersymmetry has recently been explored in ref. 3. Here there is no degeneracy at all, but rather a tight alignment between the quark and squark mass matrices. This is done in the context of a more complete theory of fermion masses with many stringy features.

5. Symmetries and String Theory

String theory has much to say about the question of symmetries which might plausibly appear in a low energy effective field theory:

1. Global vs. local continuous symmetries: It has long been argued that it doesn't make much sense to impose global symmetries in field theory. In string theory, this prejudice takes on the status of a theorem: one can show that string theory possesses no global continuous symmetries.^[5]
2. Discrete symmetries: String models often exhibit a rich structure of discrete symmetries.^[21] These can often be thought of as "gauge symmetries," e.g. coordinate or gauge transformations in some higher dimensional theory.
3. Matter multiplets in string theory do not typically have the structure of conventional grand unified theories.^[21] In particular, the transformation properties of fields under discrete symmetries do not correspond to those of GUT multiplets.

Discrete symmetries are of great phenomenological importance in supersymmetric model building: they are needed to forbid proton decay, and perhaps other rare processes ($\mu \rightarrow e\gamma$, etc.). They appear in many other possible extensions of the standard model, as well. It has been argued that any discrete symmetries appearing in an effective lagrangian should be gauge symmetries.^[22] Otherwise, they are likely to be spoiled by gravitational effects. It has also been stressed that discrete symmetries may be anomalous;^[23,24] this suggests anomaly constraints on discrete symmetries. In ref. 23, these constraints were enumerated assuming that the discrete symmetries were embedded in a broken continuous gauge group, and that the charges of the heavy states were integer multiples of those of the light states. This lead to a quite strong set of constraints. In general, however, only a weaker set of constraints hold; these can be understood in terms of instantons in the low energy theory.^[23,25]

In ref. 25, it was noted that in string theory there *are* often anomalous discrete symmetries. However, one can always cancel these anomalies by assigning to the (model-independent) axion a non-linear transformation law under the symmetry,

$$a \rightarrow a + 2\pi\delta. \quad (5.1)$$

(δ would be a multiple of $1/N$ for a Z_N symmetry). (This possibility had been suggested in ref. 23.) Such a transformation law means, of course, that the discrete symmetry is spontaneously broken (at a scale of order M_p). However, this observation has another consequence: in perturbation theory, in such cases, there is an unbroken, *global*, anomalous discrete symmetry. After all, perturbation theory exhibits an unbroken (spontaneously or explicitly) discrete symmetry.

It is perhaps helpful to give a simple example of this phenomenon (which was discussed in ref. 25). Such an example is provided by the compactification of the heterotic string on the Calabi-Yau manifold associated with the quintic hypersurface in CP^4 , discussed, for example, at some length in the textbook of Green, Schwarz and Witten.^[26] At certain points in the moduli space, this model possesses a freely-acting $Z_5 \times Z_5$ symmetry. In the textbook treatment, one mods out by this symmetry, including Wilson lines, to obtain a model with a low number of generations and a reasonable gauge group. For our purposes (despite our title) we are not concerned if the number of generations happens to be large. We can mod out by one of the Z_5 's, corresponding to rotating the coordinates, Z_a , of CP^4 , by phases:

$$Z_a \rightarrow \alpha^a Z_a$$

where $\alpha = e^{2\pi i/5}$. This is still freely acting; this means that we don't have to worry about the appearance of massless particles in twisted sectors.

The main virtue of this choice is that it leaves over a set of R -symmetries. For definiteness, consider the symmetry under which $Z_1 \rightarrow \alpha Z_1$. Under this symmetry, the gluinos transform by a phase $\alpha^{-1/2}$. Now we can include a Wilson line without breaking this symmetry. For example, we can include a Wilson line in the “second” E_8 (the one which is unbroken in the absence of the Wilson line), described by:

$$a = \frac{1}{5} (1, 1, 2, 0, 0, 0, 0, 0). \quad (5.2)$$

(I am using the notation which is standard in the orbifold context). By itself, this choice is not modular invariant, but this is easily repaired by including a Wilson line in the first E_8 as well. In the second E_8 , there are two unbroken gauge groups. It is easy to determine the effects of instantons by simply examining $SU(2)$

subgroups of these. One finds that instantons of the first group have four gluino zero modes, while instantons of the second have 24. Thus assigning to the axion a transformation law

$$a \rightarrow a + \frac{12\pi}{5} \quad (5.3)$$

(the reason for writing 12 rather than 2 will become clear shortly) cancels both anomalies.

This observation is potentially of great importance for model building. It suggests that it is reasonable to impose anomalous, global discrete symmetries. For example, one might impose a symmetry which forces $m_u = 0$ in order to solve the strong CP problem. (Recall that the discrete symmetries of string theory don't have the structure of those of conventional GUT's.)

These anomalous discrete symmetries are interesting from at least one other viewpoint. They should permit one to make *exact non-perturbative* statements about string dynamics.^[27] The reason is simple. The axion and dilaton together make up the complex scalar in a supermultiplet, usually denoted S ,

$$S = \frac{1}{g^2} + ia. \quad (5.4)$$

Given the axion transformation law of eqn. (5.1), $W(S)$ must take the form

$$W(S) = \sum_n C_n(M) e^{-n/\delta}.$$

This is precisely the behavior one encounters in the gluino condensation scenario. In these cases, one can argue that there can be no stringy non-perturbative effects stronger than the effects observed in the low energy field theory! The main subtlety in this argument lies in the determination of δ . δ cannot be determined unambiguously from the anomaly considerations described above; typically one can add a constant of the form $2\pi r$, where r is an integer. For example, had we chosen 2 rather than 12 in eqn. (5.3), the superpotential of gluino condensation would not have been invariant. This issue will be discussed further in ref. 27.

Can one generalize this? The discussion here is close in spirit to discussions of "dilaton duality" which have appeared recently in the literature.^[7] I suspect that some version of this duality is indeed correct; however it is difficult to establish it by the sort of arguments which have been employed here. (See the talks by Joanne Cohn and John Schwarz at this meeting.) These issues will also be discussed in ref. 27.

6. String Theory and the θ Puzzle

There are three known solutions to the strong CP problem.

1. $m_u = 0$. This solution may be consistent with current algebra.^[28] As we have noted in the previous section, this is a prediction which could quite plausibly emerge from string theory.
2. Peccei-Quinn symmetries and axions: here one needs a global symmetry to a high degree of approximation in perturbation theory.
3. Spontaneous CP violation: in theories with an exact CP symmetry, the bare θ is zero. The problem is then to arrange that θ be small at tree level after symmetry breaking, and that radiative corrections be sufficiently small.^[10,11]

String theory has interesting statements to make about all three possibilities. To consider these, we should first understand the status of CP as a symmetry of string theory. Strominger and Witten were probably the first to comment on CP in string theory.^[29] These authors noted that certain Calabi-Yau compactifications possess unbroken CP at points in their moduli spaces. CP , then, is a symmetry of the classical theory which can be spontaneously broken by vev's for moduli. It is not hard to see that in these cases, CP is a good symmetry of perturbation theory, since it corresponds to a two-dimensional symmetry which commutes with the BRS operator.

What about non-perturbatively? For example, there has been speculation on the possible existence of non-perturbative parameters in string theory. Could some of these exist and violate CP (θ , after all, is the quintessential non-perturbative parameter)? While we don't understand much about non-perturbative string theory, the answer to this question is a definite "no." CP turns out to be a gauge symmetry in string theory, a combination of general coordinate and gauge transformations in the higher dimensional space.^[30,31]

To see this, consider the $O(32)$ or $E_8 \times E_8$ heterotic strings in ten dimensions. In ten dimensions, P is not a symmetry of either theory. What one would like to call C has the effect of changing the signs of all of the lattice momenta, while acting trivially on spinors (as a consequence of GSO). This change of signs is a symmetry of the lattice; in fact, it is a gauge transformation. In $O(32)$, this transformation acts on a 32 component vector by the matrix:

$$\Lambda = \begin{pmatrix} i\sigma_2 & 0 & \dots \\ 0 & i\sigma_2 & \dots \\ \dots & \dots & \dots \end{pmatrix}. \quad (6.1)$$

A similar transformation works in E_8 .

Now compactify to four dimensions on an ordinary torus. In this theory, it is well-known that P is a symmetry. Where did this come from? Denote the compactified coordinates by y^a , $a = 1 \dots 6$, and the uncompactified coordinates by x^μ . The transformation

$$x^i \rightarrow -x^i \quad y^2 \rightarrow -y^2 \quad y^4 \rightarrow -y^4 \quad y^6 \rightarrow -y^6 \quad (6.2)$$

is a good space-time symmetry (it commutes with GSO). From the perspective of ten-dimensions, it is part of the proper Lorentz group. Thus for these compactifications, CP is a product of an ordinary transformation and a general coordinate transformation (with a little work, you can check that this transformation has the expected action on spinors). Other compactifications, such as Calabi-Yau compactifications, typically preserve this product of symmetries at some points in the moduli space.

CP is thus special. It can't be broken by no-perturbative effects (instantons, wormholes, etc.); non-perturbative, CP -violating parameters cannot arise in string theory, and in particular there can be no bare θ (e.g. in the $E_8 \times E_8$ theory). Thus all CP -violating effects in string theory are, in principle, calculable.

An obvious question is whether one can use this to implement the spontaneous CP -violation solution to the strong CP -problem. A few observations are important. First, to accomplish this, one wants CP broken at a scale well below M_p .^[30] If Λ is the scale of CP violation, even if one suppresses low dimension operators contributing to θ , there will inevitably be operators of high dimension, whose contribution will be of order $(\Lambda/M_p)^n$. Thus one probably wants to break CP at a scale not larger than 10^{10} GeV, or so (assuming that the leading contributions are due to dimension five operators). Otherwise, the effective θ is almost certainly of order one.^[30]

E_6 models, such as those which arise in $(2, 2)$ compactifications of string theory, provide a rather natural setting for the Nelson-Barr mechanism.^[32] Under $O(10) \times U(1)$, the 27 of E_6 decomposes as

$$27 = 16_{-1/2} + 10_1 + 1_{-2}. \quad (6.3)$$

Here the 16 contains an ordinary generation of quarks and leptons and an additional right-handed neutrino, \mathcal{N} , while the ten contains a vectorlike (with respect to the standard model group) set of doublets, H_1 and H_2 , and a vectorlike set of charge $-1/3$ quarks, q and \bar{q} . We will denote the $O(10)$ singlet by S . The $SU(3) \times SU(2) \times U(1)$ singlet fields, \mathcal{N}_i and S_i can naturally obtain vev's of order^[33]

$$M_{INT} = \sqrt{m_W M_p} \approx 10^{11} GeV. \quad (6.4)$$

Moreover, there is typically a range of soft-breaking parameters for which the

$\langle S_i \rangle$ are real, while the $\langle \mathcal{N}_i \rangle$ are complex. The allowed couplings in the superpotential in these models are of the form

$$W = \mathcal{N}q\bar{d} + Sq\bar{q} + H_1Q\bar{u} + H_2Q\bar{d}. \quad (6.5)$$

The fermion mass matrix then has the Nelson-Barr structure:

$$m_F = \begin{pmatrix} \Gamma H_2 & \gamma \mathcal{N} \\ 0 & \mu \end{pmatrix}. \quad (6.6)$$

Automatically, $\arg \det m_F = 0$.

At tree level, there are many couplings which must be suppressed for all of this to work. For example, it is necessary to avoid phases in the Higgs potential, and to suppress couplings which would lead to phases in the S vev's. This can be accomplished by discrete symmetries. In loops, however, more serious problems arise which cannot be dealt with so easily. Early studies of loop corrections to θ showed that many contributions can be suppressed.^[34] However, these analyses assumed that squarks are precisely degenerate. If one relaxes the assumption of exact degeneracy and examines the various corrections one finds that the smallness of θ sets stringent requirements. In particular, diagrams correcting the d quark mass lead to a requirement that $\frac{\delta m^2}{m_{susy}^2} < 10^{-5}$; diagrams contributing to the gluino mass require that certain phases be aligned to one part in 10^7 . Neither of the schemes we have discussed above (flavor symmetries or dilaton driven supersymmetry) seem likely to produce anything like this degree of degeneracy. Perhaps these constraints can be satisfied as a consequence of symmetries. But at the least a much more elaborate symmetry structure is required to implement the Nelson-Barr scheme in the framework of supersymmetry than has been considered to date.

7. Axions in String Theory and Elsewhere

The underlying idea to the axion solution to the strong CP problem is to postulate a ‘‘Peccei-Quinn symmetry’’ under which the axion transforms as

$$a \rightarrow a + f_a \delta. \quad (7.1)$$

This solution, however, suffers from serious problems, which, like the Barr-Nelson solution discussed above, call its plausibility into question.^[9]

The basic problem is simple, and should be ‘‘obvious’’ to string theorists. In string theory, and presumably in any fundamental theory, there are no global, continuous symmetries. Approximate global symmetries, if they exist, might arise accidentally, as an accidental consequence of the structure of low dimension terms in the low energy effective lagrangian (like B and L in the standard model).

To see the difficulty, recall that the usual axion potential has the form

$$V = -Nm_\pi^2 f_\pi^2 \cos(a/f_a). \quad (7.2)$$

Suppose that \mathcal{O}_n is the lowest dimension operator which breaks the Peccei-Quinn symmetry; its dimension is $n+4$. Then the symmetry-breaking lagrangian has the form

$$\mathcal{L}_{SB} = \frac{\gamma}{M_p^n} \mathcal{O}_n. \quad (7.3)$$

This gives rise to an axion potential, on dimensional grounds, of the form

$$\delta V = \frac{\gamma f_a^{n+1}}{M_p^n} a(x). \quad (7.4)$$

If $f_a \sim 10^{11} \text{ GeV}$, then requiring $\theta = a/f_a < 10^{-9}$ gives $n > 7$, i.e. it is necessary to suppress operators up to dimension 11 in order to obtain a sufficiently good symmetry!

But string theory always has an axion in perturbation theory.^[35] This axion arises from the antisymmetric tensor field, $B_{\mu\nu}$, which in four dimensions is equivalent to a scalar. Moreover, the decay constant of the axion is of order M_p . From the perspective of the argument given above, this is highly suprising. It can be understood in a variety of ways. First, the couplings of the antisymmetric tensor are governed by a gauge principle. This insures that in perturbation theory, $B_{\mu\nu}$ enters the effective lagrangian only through the gauge-invariant field strength $H_{\mu\nu\rho}$.^{*} Alternatively, one can understand this result directly in terms of the string perturbation expansion. At $k=0$, the vertex operator for the axion is a total derivative.

While the existence of this axion is rather suprising from a field theoretic perspective, it is not at all clear that this field can solve the strong CP problem. First, its decay constant is of order M_p , which poses cosmological difficulties. Second, if there is any sort of hidden sector gauge group, it is likely to give mass to this axion.

Thus, if the string theory solves the strong CP problem by the Peccei-Quinn mechanism, another axion must arise by accident as a consequence of a continuous

^{*} One might naturally ask how the axion potential due to non-Abelian gauge interactions is consistent with this symmetry. In fact, it is. Moreover, it is not hard to understand why only non-perturbative effects give rise to this mass. I thank Renata Kallosh, Lenny Susskind for discussions of this and related matters, which will appear elsewhere.

or a *discrete* gauge symmetry. This latter possibility was already considered some time ago in refs. 36 and 37. These authors noted, as we have mentioned earlier, that it is quite natural to obtain a scale $f_a \sim 10^{11} GeV$ in string theory. For example, the field S of eqn. (6.3) can appear in the superpotential only in a restricted way. Because there is no mass for this field, and because of its gauge quantum numbers, the leading term which can give rise to a potential for the field S and the corresponding field \bar{S} which might arise from the $\overline{27}$, is

$$W = \frac{1}{M_p^2} S^2 \bar{S}^2. \quad (7.5)$$

If the soft breaking terms include terms of the form

$$V_{soft} = -m_{3/2}^2 |S|^2 + \dots \quad (7.6)$$

then $S^2 \sim m_{3/2} M_p$. In order that there be an approximate symmetry, one needs to forbid many operators in W and V_{soft} . This is possible with rather simple discrete symmetries.^[36,37,38]

8. Summary

We have touched here on a large number of topics. There are a few points which I hope you will take away from this talk:

1. Kaplunovsky and Louis have exhibited one is perhaps the first example of something which might cautiously be referred to as a string prediction. In particular, string theory offers a simple option for explaining squark and slepton degeneracy; this framework makes a series of strong predictions.
2. String theory offers several lessons for model building.
 - a. Discrete symmetries in string theory can be global and anomalous (relevant for $m_u = 0$?); discrete symmetries in string theory don't look like those from conventional GUT's.
 - b. CP is an exact, gauge symmetry of string theory. But even though CP violation is spontaneous and the "bare" θ vanishes, it is difficult to solve the strong CP problem without axions or $m_u = 0$.
3. String theory suggests alternative approaches to the problem of squark degeneracy, based on both continuous and discrete flavor symmetries.

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